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Design of Microwave Filters by Sine-Plane Approach

KHEE K. PANG

Abstract—**Using a new sine-plane approach [7], an easy-to-use design procedure for microwave filters is developed. The design formulas are very simple (Tables I–III) and are valid for filters of wide bandwidths (Section V). Furthermore, the new design offers many advantages over other presently available designs.**

I. Introduction

Microwave filters can be designed using two different approaches. They can be designed by approximation equations [1]–[3], and they can be designed by exact synthesis methods [4], [5]. Both approaches have their own merits. Explicit formulas are given in Cohn [1], Matthaei [2], and Cristal [3]; their design procedures are therefore easy to use. The design method presented by Wenzel [4] and Mumford [5] is exact, but this is achieved at the expense of greater numerical complexity.

With reference to the first approach, i.e., designing filters by approximation equations, Dishal [6] raised the following question: Why should one waste the space to put in rods #0 and #$(n+1)$, the only purpose of which is to properly couple the resistive generator and resistive load to the input–output resonances, respectively? This question has not been answered satisfactorily for parallel-coupled filters of wide bandwidths.

Using the new sine-plane approach presented in the companion paper [7], a new set of design equations will be derived in this paper. Apart from dissolving the objection raised above, the new design equations offer other advantages. These are discussed in the text of the paper.

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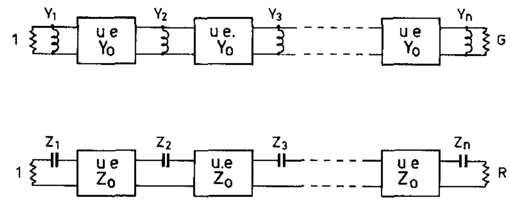

Fig. 1. Basic bandpass networks.

II. Theory

The theory derives mainly from the graphical transformation technique described in [7]. Through a series of transformations, it will be shown that microwave bandpass filter networks can be identified with lumped prototype filters of standard designs.

Consider the two basic bandpass filter structures in Fig. 1.[1] They are shown in Richards' λ-plane presentation,[2] and thus implicitly assumed that the networks consist of open-circuit stubs, short-circuit stubs, and unit transmission lines, all of which have the same electrical length. Note that all connect-

[1] It can be shown that the interdigital filter and parallel-coupled filter are equivalent to one of the two network structures.

[2] $\lambda = \tanh \tau p$, where τ is the time taken for a pulse to traverse the unit length of transmission line, and $p = \sigma + j\omega$ is the complex frequency. Later, s and c will be used to denote $\sinh \tau p$ and $\cosh \tau p$, respectively.

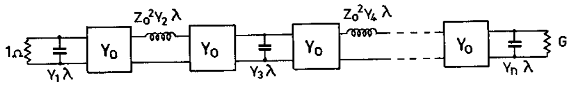

Fig. 2. Low-pass equivalent for n odd. The last element is a series inductor of value $Z_0^2Y_n\lambda$ if n is even.

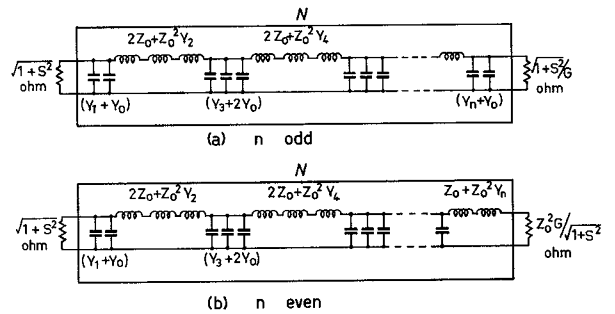

Fig. 3. S-plane equivalent of low-pass filter.

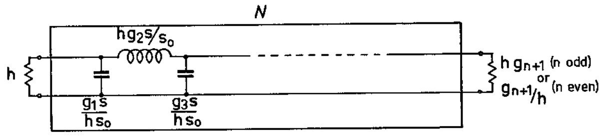

Fig. 4. Frequency-scaled and impedance-scaled low-pass prototype filter.

TABLE I

DESIGN EQUATIONS FOR SHUNT SHORT-CIRCUITED STUB FILTERS OF FIG. 1(a)

Nomenclature

n	Number of shunt stubs.
w	Fractional bandwidth.
g_i	Element values of prototype network.
G	Terminating conductance [see Fig. 1(a)].
h	Scaling factor. Recommended value $h=\sqrt{c_0}$ [see (3)].
s_0	$=\sin(w\pi/4)$.
c_0	$=\cos(w\pi/4)$.

For n even

Connecting elements

$Y_0=\sqrt{Gg_{n+1}}/h$

End stubs

$Y_1=g_1/(hs_0)-Y_0$

$Y_n=hg_nY_0^2/s_0-Y_0$

Intermediate stubs

$Y_i=g_i/(hs_0)-2Y_0$, for $i=3, 5, \cdots, n-1$

$Y_i=hg_iY_0^2/s_0-2Y_0$, for $i=2, 4, \cdots, n-2$

For n odd

Connecting elements

Y_0 arbitrary, $G=1/g_{n+1}$

End stubs

$Y_1=g_1/(hs_0)-Y_0$

$Y_n=g_n/(hs_0)-Y_0$

Intermediate stubs

$Y_i=g_i/(hs_0)-2Y_0$, for $i=3, 5, \cdots, n-2$

$Y_i=hg_iY_0^2/s_0-2Y_0$, for $i=2, 4, \cdots, n-1$

ing elements are identical. This would not restrict the generality of filter characteristics, as will be clear later.

Fig. 2 shows the "low-pass" equivalent of the network in Fig. 1(a). These two equivalent networks have identical frequency characteristics, but are displaced from one another by an amount f_0 along the frequency axis, where $f_0=1/4\tau$ is the quarter wavelength frequency. The low-pass equivalent can be derived either by the procedure described in Cristal [8], or in Kuroda [9]. The other low-pass equivalent of the network in Fig. 1(b) is a dual network of that in Fig. 2. Therefore, it will be omitted here for brevity.

The network in Fig. 2 is next presented in an s-plane equivalent form. Using the procedure described in Section IV of [7], the lumped network of Fig. 3 is obtained. Note that the $\sqrt{c}$-transformer created in the process is eliminated by showing the two terminating resistors, $\sqrt{1+s^2}$ and $\sqrt{1+s^2}/G$ (or $Z_0^2G/\sqrt{1+s^2}$) as frequency-dependent resistors.

Except for these terminating resistors, the s-plane equivalent is identical to a lumped ladder network shown in Fig. 4 in every other respect. Let us disregard the terminations for the moment and equate the element values of the two lossless two-ports. The results are listed in Table I. Using the lumped prototype filter as the basis, Table I enables us to derive the element values of the corresponding microwave bandpass filter. It will be shown later that the bandpass filter has nearly the identical frequency characteristic as its lumped prototype filter.

III. Mismatch at Terminations

In the s-plane presentation of Fig. 3, the two frequency-dependent resistors at the terminations are similar to the image impedance of a k-section in the classical image-parameter filter design, except for one important distinction. The classical image impedance is resistive in the passband, and becomes reactive in the stopband; whereas the $\sqrt{1+s^2}$ resistor remains resistive throughout the entire frequency range.

If we consider the low-pass prototype filter in Fig. 4 as a lossless two-port N "matched" at both terminations, then the low-pass equivalent in Fig. 3 can be regarded as the same filter network N having mismatched terminations. Like the classical image-parameter filter, it is these frequency-dependent resistors that distort its predicted frequency characteristics.

If the $\sqrt{1+s^2}$ resistor has a dominant value of h over the frequency range of interest,[3] the reflection caused by the slight mismatch should be small enough to have no appreciable effect on the overall frequency characteristic. The low-pass network in Fig. 3 and the prototype filter in Fig. 4 should then have the same frequency characteristic. At frequency when the $\sqrt{1+s^2}$ resistor is exactly equal to h, i.e., when

$$\sqrt{1+s^2}=\cos\tau\omega=h \tag{1}$$

the microwave filter is matched at both terminations, and a perfect matching is said to have been established.

IV. Optimum Impedance Level h

As the impedance level h of the prototype network can be varied or arbitrary without affecting its frequency responses, the perfect matching condition can be established at any set frequency.

If we were to set $h=1$, perfect matching would occur at the center of the passband. The design would then be similar to Cohn's [1] in this respect. If we were to set $h=c_0$, perfect matching would occur ot the edges of the passband and the design would then be similar to Cristal's [3].

Instead, a different value of h will be chosen to the "best" advantage. In the following, the criterion of the Chebyshev

[3] This normally includes the passband and its vicinity.

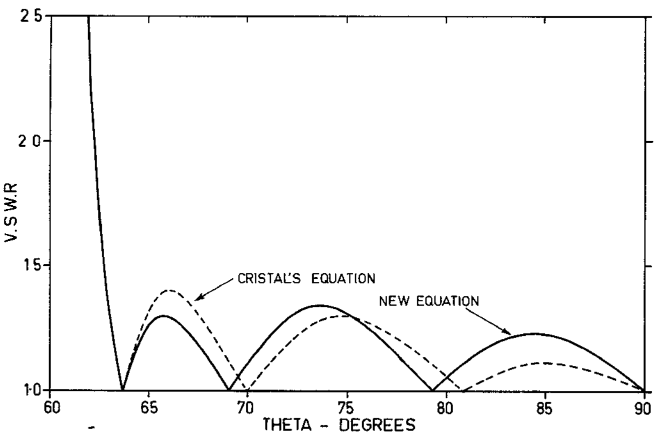

Fig. 5. Comparison of computer responses derived from prototype filter having $n=7$, VSWR$=1.36$, $w=0.6$.

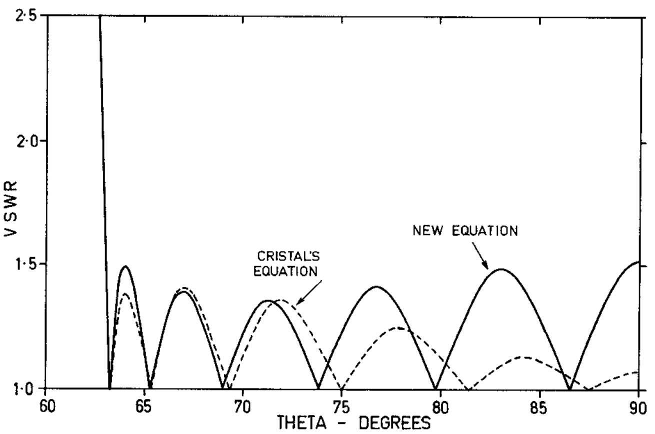

Fig. 6. Comparison of computer responses derived from prototype filter having $n=12$, VSWR$=1.36$, $w=0.6$.

filter will be invoked and the value of h chosen to give a minimum passband ripple.

Let us first assume that the terminating resistors are frequency independent, the Product Theorem[4] [14] yields the following result

$$\bar{r} = r_p \max_{c_0 \le c \le 1}\left[\left(\frac{c}{h}\right)^2, \left(\frac{h}{c}\right)^2\right] \tag{2}$$

where $\bar{r}$ denotes the maximum VSWR of the microwave filter in the passband.

r_p denotes the maximum VSWR of the prototype network in the passband and $\max_{c_0 \le c \le 1} [(c/h), (h/c)]$ represents the VSWR of the mismatches at the terminations over the passband.

Minimizing r with respect to h in (2), we obtain

$$h = \sqrt{c_0} \tag{3}$$

which corresponds to a maximum VSWR of r_p/c_0. This value of VSWR can only be reached at the center and at the edges of the passband, where maximum mismatch occurs.

[4] Strictly speaking, the Product Theorem as given in [14] is not applicable to the present problem. However, the result in (2) can be proven readily by an optimization procedure [11], [15]. Also, (3) is not a true optimum solution as the assumption that the terminating resistors are frequency independent is not true. However, as the relaxation of any constraint relationship can only produce a higher value of r in the optimization process, (4) is still valid.

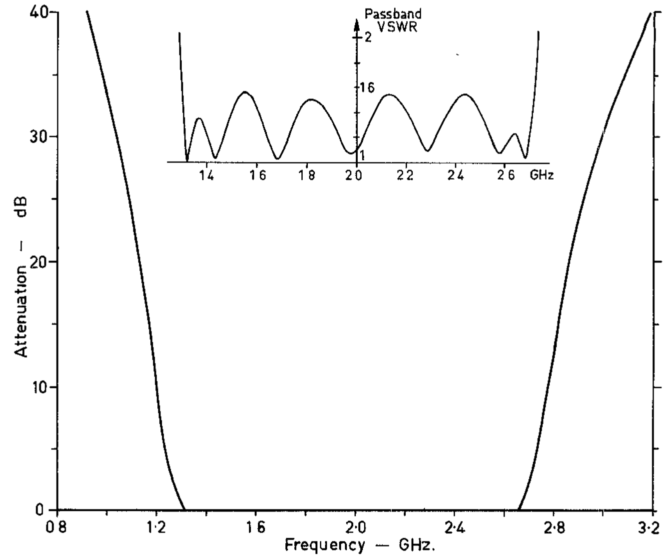

Fig. 7. Computed filter response. The filter is designed from the prototype network with $n=7$, $w=0.7$, and bandpass ripple 0.25 dB (VSWR$=1.62$).

For even-order (n even) filters, it can be shown that the choice of h in (3) will yield a maximum VSWR of r_p/c_0 at the center of the passband (see Fig. 6). For odd-order filters, however, this value may not be reached (see Fig. 5). We can therefore write

$$\bar{r} \le r_p/c_0, \qquad \text{for } h = \sqrt{c_0}. \tag{4}$$

V. Amplitude Response of the Filters

Once the value of h is chosen, the filter network as defined in Table I is completely specified. The remaining task then is to assess how good the new design equation is. For this purpose, the amplitude characteristics of the new filters are compared against Cristal's in Figs. 5 and 6 for typical cases of n odd and n even. Among the approximation design equations presently available, Cristal's equations appear to give the best approximation characteristics [3].

Fig. 5 shows that the new filter has a lower VSWR than that of Cristal's. Fig. 6, on the other hand, shows that the new filter has a higher VSWR. All in all,[5] it was found that the degree of approximation offered by the two different methods is about equal.

In Fig. 7, the new filter response is compared against the prototype filter response. The impedance scaling factor h is set at one, thus creating a maximum number of possible mismatches at the edges of the passband. The fractional bandwidth is set to 70 percent, which is more than an octave bandwidth. Under this adverse condition, the new filter still closely follows the ripple characteristic of its prototype network.

VI. Experimental Example

To illustrate the design procedure, a 7-stub bandpass filter of 70-percent fractional bandwidth will be designed step by step as follows.

[5] When evaluating the frequency response of a Chebyshev filter, the selectivity in the stopband must also be taken into consideration. In the two examples shown here, the steepest slopes (in dB/degree) of the new filters in the topband are both slightly higher than those of the corresponding Cristal's filters.

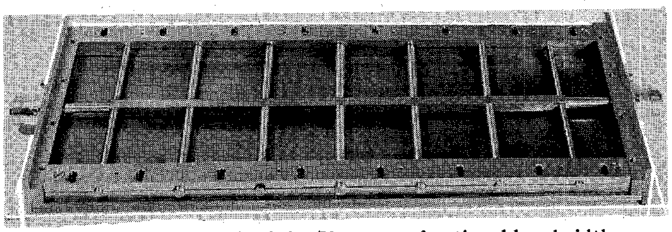

Fig. 8. Photograph of the 70-percent fractional bandwidth filter with cover plate removed.

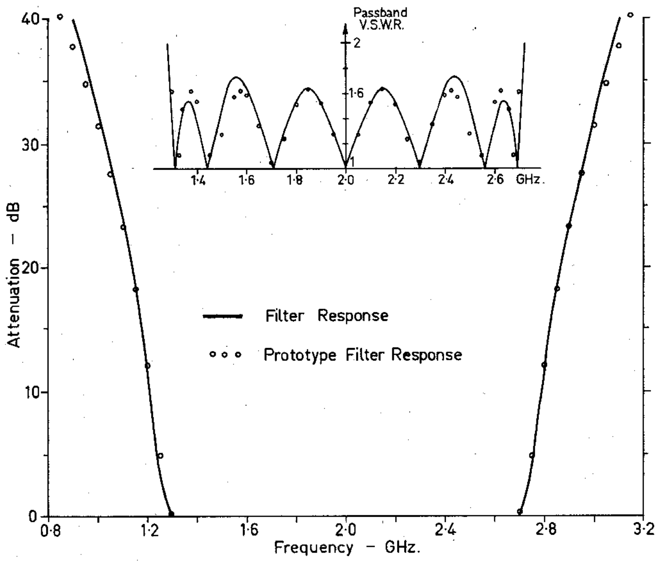

Fig. 9. Measured filter response.

Assume that the design calls for a Chebyshev prototype filter of 1/4-dB (VSWR = 1.62) ripple in the bandpass and that h is to be chosen as 1.[6] This will result in a maximum ripple height of 0.325 dB (VSWR = 1.74) in the actual bandpass filter response, as was indicated in the computed characteristic curves of Fig. 7.

Table I indicates that the choice of Y_0 is arbitrary. In order to reduce the spread in element values, this added degree of freedom is utilized to set $Y_2 = Y_3 = Y_5 = Y_6$, which results in $Y_0^2 = g_3/g_2$.

The above specifications produce the following prototype element values [13].

$$g_1 = g_7 = 1.4468 \qquad g_2 = g_6 = 1.3560$$
$$g_3 = g_5 = 2.3476 \qquad g_4 = 1.4689.$$

When substituting into the formulas in Table I, these values in turn yield the following set of normalized element values for the stub filter.

$$Y_0 = 1.3158$$
$$Y_1 = Y_7 = 1.4533$$
$$Y_2 = Y_3 = Y_5 = Y_6 = 1.8614$$
$$Y_4 = 2.2356.$$

[6] h should have been chosen to be $\sqrt{\cos 31.5°}$. The experimental work was done before the author realized that the impedance level can be adjusted to a better advantage.

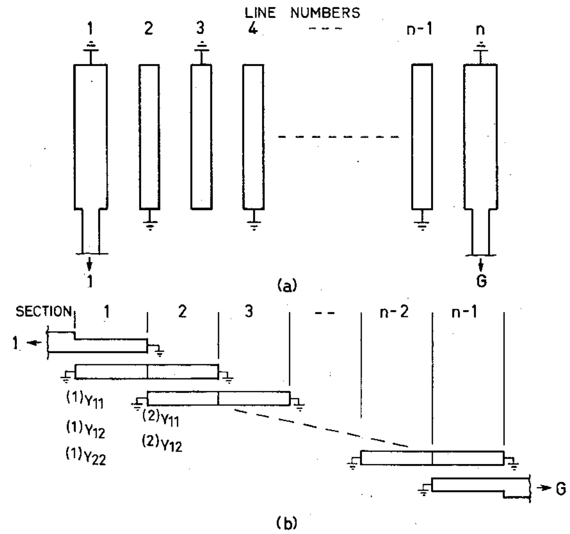

Fig. 10. (a) Interdigital filter. (b) Parallel-coupled half-wave short-circuited resonator filter.

TABLE II

DESIGN EQUATIONS FOR INTERDIGITAL FILTERS SHOWN IN FIG. 10(a)

Nomenclature

See Fig. 10(a) and Table I

Mutual capacitances (normalized to v^{-1}, where v is the velocity of propagation). All mutual capacitances are identical, i.e., $C_{i,i+1} = C_0$, $i = 1, 2, 3, \cdots, n-1$

If n even, $C_0 = \sqrt{Gg_{n+1}}/h$

If n odd, C_0 arbitrary and $G = 1/g_{n+1}$

Self-capacitances[a] (normalized to v^{-1})

$C_{ii} = g_i/(hs_0)$, $\quad i = 1, 3, 5, \cdots, n$

$C_{ii} = h \cdot C_0^2 g_i/s_0$, $\quad i = 2, 4, 6, \cdots, n$

[a] The running index i terminates either at n or at $n-1$, the integer before it.

TABLE III

DESIGN EQUATIONS FOR PARALLEL-COUPLED HALF-WAVE SHORT-CIRCUITED RESONATOR FILTER AS SHOWN IN FIG. 10(b)

Nomenclature

See Fig. 10(b) and Table I

Mutual admittances. All mutual admittances are identical, i.e., ${}^{(i)}Y_{12} = Y_0$, $i = 1, 2, 3, \cdots, n-1$

If n even, $Y_0 = \sqrt{Gg_{n+1}}/h$

If n odd, Y_0 arbitrary and $G = 1/g_{n+1}$

Self-admittance[a]

${}^{(i)}Y_{11} + {}^{(i-1)}Y_{22} = g_i/(hs_0)$ $\quad i = 1, 3, 5, \cdots, n$

${}^{(i)}Y_{11} + {}^{(i-1)}Y_{22} = hY_0^2 g_i/s_0$ $\quad i = 2, 4, 6, \cdots, n$

where ${}^{(0)}Y_{22} = {}^{(n)}Y_{11} = 0$ for the end sections

For intermediate sections

${}^{(i)}Y_{11} = {}^{(i-1)}Y_{22}$, $\quad (i = 2, 3, 4, \cdots, n-1)$ is recommended

[a] The running index i terminates either at n or at $n-1$, the integer before it.

Fig. 8 shows the realization of this filter in slabline form. The center piece is a continuous bar of rectangular cross section. The shunt stubs are of circular cross section. Their numbers are doubled to reduce junction size and also to provide mechanical rigidity.

The passband VSWR and stopband attenuation characteristic of this filter were measured and the results depicted in

Fig. 9. With the exception of one smaller ripple and slower rate of cutoff at 3 GHz–30-dB region, the experimental result is in very close agreement with the theoretical prediction shown in Fig. 7.

VII. Coupled Filters

The design equations of Table I can be modified to include coupled filters such as those shown in Fig. 10. This can be done readily by the graphical transformation technique [12], and the results are presented in Tables II and III.

VIII. Conclusion

The design formulas presented in Tables I–III have the following advantages over other existing approximate design formulas.

1) The new design formulas are simpler.

2) Two fewer sections are required for the parallel-coupled filters.

3) The identical coupling parameters (Y_0) in the filter structure may offer some mechanical advantages in the physical realization of the filter.

4) The worst VSWR of the filter in the passband can be predicted (4) and precorrected if necessary.

The comparative ease with which these new design equations were derived also demonstrated the effectiveness of the new approach presented in the companion paper [7].

Acknowledgment

The author wishes to thank the reviewers for their helpful suggestions.

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Design of Acoustic Surface-Wave Devices Using an Admittance Formalism

ALAN S. BURGESS AND PETER H. COLE

***Abstract*—The advantages of an admittance formalism for the derivation of performance characteristics of transversal filters and one-port information stores using acoustic surface-wave delay lines are described. An expression for the transadmittance between transducer pairs in the weak-coupling approximation is derived using a normal mode theory. The formulation is found to give good agreement with measurements of the passband response of a wide-band logarithmically frequency-tapered transducer pair on *YX*-quar tz A brief discussion of the limitations of the model is included.**

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I. Introduction

THE ART of signal processing by means of acoustic surface-wave devices depends in large measure on the exploitation of the characteristics of multitapped delay lines in the synthesis of two-port transversal filters. Multitap delay lines in which all the transducers are connected in parallel to form a one-port device also find application in the field of information storage and encoding in that they are one-port